13 November 2015, CTTC, Castelldefels

From Network Coding to Uncoordinated Multiple Access

Stephan Pfletschinger

Institute for Communications and Navigation
German Aerospace Center, DLR

European School of Antennas

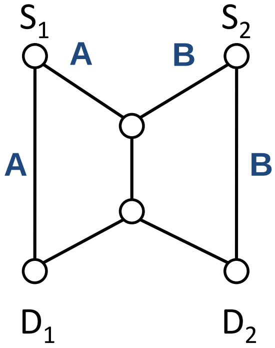
WIRELESS NETWORKS: FROM ENERGY HARVESTING TO INFORMATION PROCESSING

9-13 November 2015
Castelldefels (Barcelona)

Outline

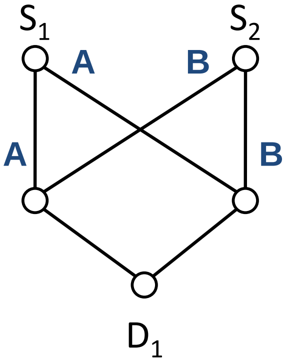
- Network Coding
  - The basic example
- Physical Layer (Wireless) Network Coding
  - Two-way relaying
  - Lattice coding
  - Joint network and channel coding
- Uncoordinated Multiple Access
  - Variations of slotted ALOHA
  - Application of network coding

Network Coding: The Butterfly Network



- Sources S_1, S_2
- Packets A, B
- Destinations D_1, D_2
- Objective: transmit both packets to both destinations

Another network

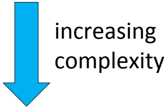


- Objective: transmit both packets to destination
- What should be transmitted on the cross-connections?

(Physical Layer) Network Coding

- Algebraic network coding:
  - formulated for signals in $\mathbb{F}_q$
  - constant, fixed connections (wires) between nodes
- Wireless (physical layer) network coding:
  - transmitted and received signals in wireless channels are real or complex-valued
  - broadcast nature of wireless channel
  - signals combine in the air

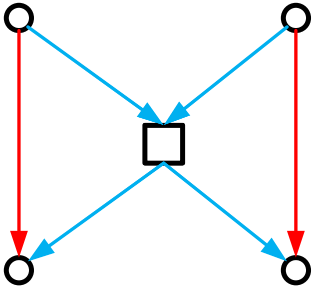
- Role of physical layer
  1. Point-to-point bit pipe
  2. Multiple-access, broadcast channel
  3. Network, including interference



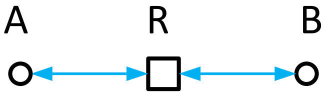
DLR

Two Canonical Examples for (Wireless) Network Coding

- Wireless butterfly

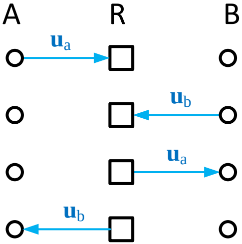


- Two-way relay channel

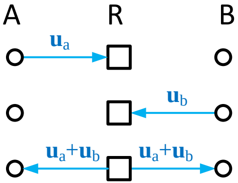


Two-Way Relay Channel: From 4 Slots to 2 Slots

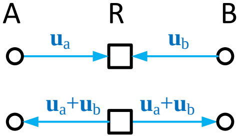
4 slots: point-to-point PHY-layer



3 slots: algebraic network coding

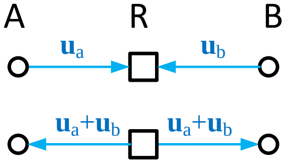


2 slots: PHY-layer network coding

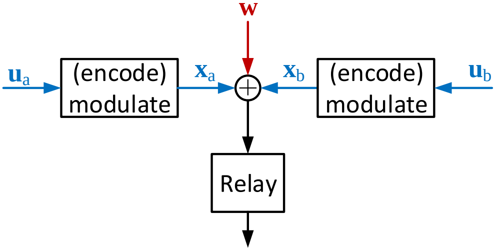


A Short Reflection on the Broadcast Phase

- Consider broadcast phase in two-slot protocol
  - Assume that the relay has the messages $\mathbf{u}_a$ and $\mathbf{u}_b$ and wants to transmit a combination of both messages
- **Question**: Transmit messages with superposition coding (like in broadcast channel) or transmit network-coded combination $\mathbf{u}_a + \mathbf{u}_b$?



Multiple-Access Phase

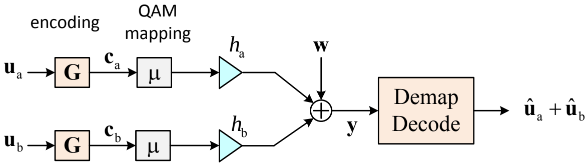


Decoding at relay:

- Relay transmits $\mathbf{u}_a + \mathbf{u}_b$ (binary sum, encoded and modulated)
- No need to receive $\mathbf{u}_a$ and $\mathbf{u}_b$ individually, only $\mathbf{u}_a + \mathbf{u}_b$ is required
- Multiple-access phase is bottleneck

Key Aspects of Network Coding and Physical-Layer NC

- Key operation of (algebraic) network coding
  - sum of packets of bits is again a packet of bits (sum in Galois field $\mathbb{F}_2$)
  - for $\mathbf{u}_\mathrm{a}, \mathbf{u}_\mathrm{b} \in \mathbb{F}_2^K$, $\mathbf{u}_\mathrm{ab} \triangleq \mathbf{u}_\mathrm{a} + \mathbf{u}_\mathrm{b} \in \mathbb{F}_2^K$
- Key idea of physical-layer network coding
  - exploit sum of signals (in $\mathbb{R}$ or $\mathbb{C}$) to decode for sum of packets (in $\mathbb{F}_2$)



  - difficulty: no direct relation between sums in finite field and in real numbers

Approaches for Two-Way Relaying

Main approaches:

1. Optimize modulation and decision regions for symbol demapping
   - uncoded approach
2. **Nested lattice codes**
   - Elegant approach to link Hamming and Euclidian spaces ($\mathbb{F}$ and $\mathbb{R}$)
   - However: fading destroys lattice
3. **Linear codes in $\mathbb{F}_2$**
   - Employ off-the-shelf channel codes
   - Fading is no problem
   - Apply the same channel code for both users
     - $\Rightarrow$ decoding for $\mathbf{u}_{ab}$ is possible with the same decoder (functional decoding)

Decoding at Relay: Basic Principle

Canonical (toy) example: Uncoded BPSK over AWGN channel

$$y = x_a + x_b + w$$

u_a	u_b	$u_a + u_b$	x_a	x_b	$x_a + x_b$
0	0	0	-1	-1	-2
0	1	1	-1	$+1$	0
1	0	1	$+1$	-1	0
1	1	0	$+1$	$+1$	$+2$

Network-Coded Modulation

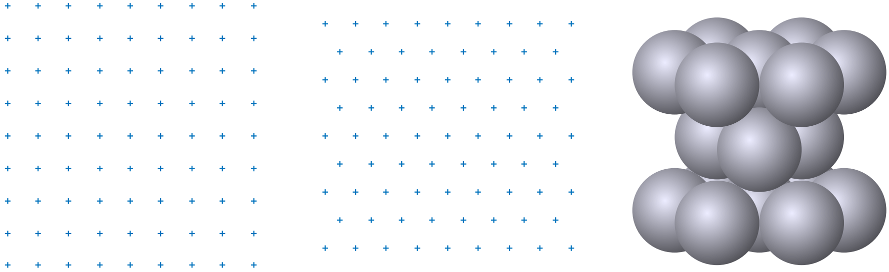
- Motivated by previous example, considers real or complex-valued modulation **symbols** [Sykora2011]
- *Hierarchical decode and forward*: defines hierarchical symbol $x_{ab} \triangleq \chi(x_a, x_b)$, for which the *exclusive law* must hold:

$$\chi(x_a, x_b) \neq \chi(x_a', x_b) \quad \forall x_a \neq x_a'$$
$$\chi(x_a, x_b) \neq \chi(x_a, x_b') \quad \forall x_b \neq x_b'$$

- Exclusive law guarantees that, given x_a, we can obtain x_b from x_{ab}
- Many literature available on related approaches, e.g. optimization of signal constellations
- Limitation: channel coding is not considered

Lattices: Some Simple Examples

- One-dimensional lattice: integers $\mathbb{Z}$ in real numbers $\mathbb{R}$
- Two-dimensional lattices: rectangular or hexagonal grid
- Three-dimensional lattice: centres of spheres in hexagonal close-pack



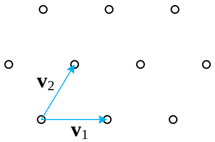
Lattices: Definition

Lattice

A *lattice* is a set of points in an N-dimensional space given by all integer combinations of a basis of up to N linearly independent vectors:

$$\Lambda \triangleq \{a_1\mathbf{v}_1 + a_2\mathbf{v}_2 + \cdots + a_N\mathbf{v}_N : \ a_1, \ldots, a_N \in \mathbb{Z}\}, \ \mathbf{v}_n \in \mathbb{R}^N$$

- The basis vectors can be gathered into a generator matrix $\mathbf{V} = [\mathbf{v}_1 \cdots \mathbf{v}_N] \in \mathbb{R}^{N\times N}$, then $\Lambda = \left\{\mathbf{Va} : \ \mathbf{a} \in \mathbb{Z}^N\right\}$



Lattice Properties and Basic Operations

- Closure under addition: $\lambda_1, \lambda_2 \in \Lambda \;\Rightarrow\; \lambda_1 + \lambda_2 \in \Lambda$
  - compare this to definition of linearity of channel codes
- Symmetry: $\lambda \in \Lambda \;\Rightarrow\; -\lambda \in \Lambda$
- Lattice quantizer: $q_\Lambda(\mathbf{x}) \triangleq \arg\min_{\lambda \in \Lambda} \|\mathbf{x} - \lambda\|$
  - finds nearest lattice point to $\mathbf{x}$
- Voronoi region of lattice point: Set of all points that quantize to that lattice point

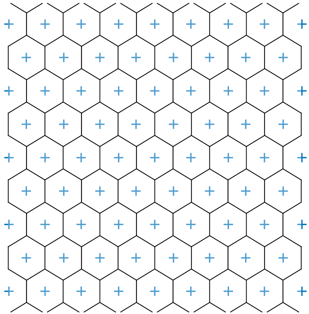
$$\mathcal{V}(\lambda) \triangleq \left\{ \mathbf{x} \in \mathbb{R}^N : \; q_\Lambda(\mathbf{x}) = \lambda \right\}$$

- Fundamental Voronoi region: Set of all points that quantize to the origin

$$\mathcal{V}_\Lambda \triangleq \left\{ \mathbf{x} \in \mathbb{R}^N : \; q_\Lambda(\mathbf{x}) = \mathbf{0} \right\}$$

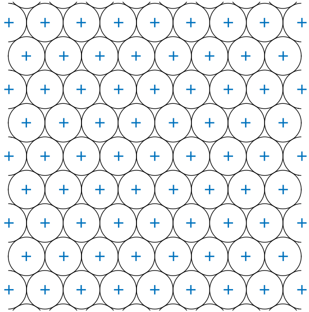
Example for Voronoi Region

- Hexagonal lattice with basis vectors $\mathbf{v}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\mathbf{v}_2 = \frac{1}{2}\begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix}$



Example for Voronoi Region

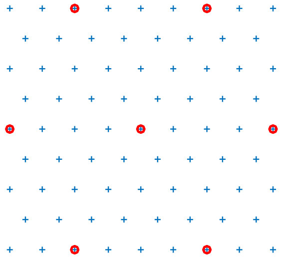
- Hexagonal lattice with basis vectors $\mathbf{v}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\mathbf{v}_2 = \frac{1}{2}\begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix}$



Nested Lattice Codes

- A coarse lattice Λ_c and a fine lattice Λ_f are nested if $\Lambda_c \subset \Lambda_f$
- A nested lattice code contains all points of the fine lattice that fall into the fundamental Voronoi region of the coarse lattice,

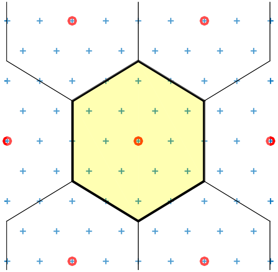
$$\mathcal{L} \triangleq \Lambda_f \cap \mathcal{V}_{\Lambda_c} \subset \mathbb{R}^N$$



Nested Lattice Codes

- A coarse lattice Λ_c and a fine lattice Λ_f are nested if $\Lambda_c \subset \Lambda_f$
- A nested lattice code contains all points of the fine lattice that fall into the fundamental Voronoi region of the coarse lattice,

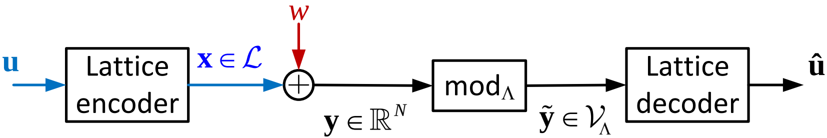
$$\mathcal{L} \triangleq \Lambda_f \cap \mathcal{V}_{\Lambda_c} \subset \mathbb{R}^N$$



DLR

Application of Nested Lattice Codes

- Modulo operator on the coarse lattice $\mathrm{mod}_{\Lambda_c}(\mathbf{x}) \triangleq \mathbf{x} - q_{\Lambda_c}(\mathbf{x})$ leads to mod-AWGN channel:



- under mod_{Λ} operation, the sum of two codewords is also a codeword:

$$\forall \mathbf{x}_1, \mathbf{x}_2 \in \mathcal{L} \;\Rightarrow\; \mathrm{mod}_{\Lambda}(\mathbf{x}_1 + \mathbf{x}_2) \in \mathcal{L}$$

$\Rightarrow$ With nested lattice codes, the sum in $\mathbb{R}$ on the channel and the sum of codewords are equivalent. [Wilson2010,Nazer2011]

Capacities and Achievable Rates

- Upper bound: Capacity of AWGN channel

$$C_{\text{upperbound}} = \frac{1}{2}\log_2(1 + \text{SNR})$$

- Nested lattice coding:

$$C_{\text{lattice}} = \frac{1}{2}\log_2\left(\frac{1}{2} + \text{SNR}\right)$$

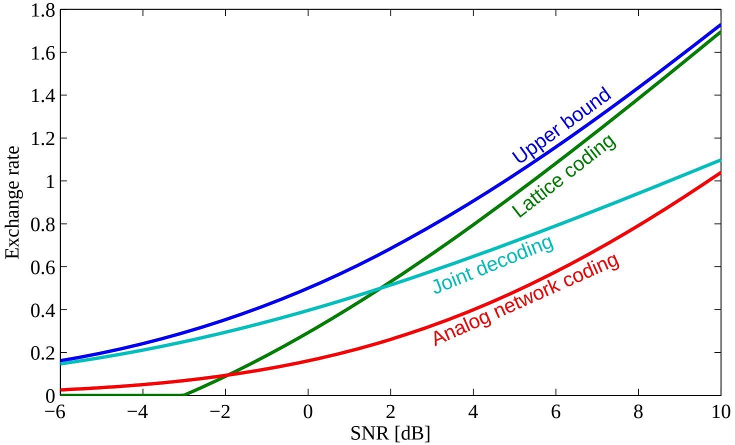
- Analog network coding:

$$C_{\text{analog}} = \frac{1}{2}\log_2\left(1 + \frac{\text{SNR}}{3 + 1/\text{SNR}}\right)$$

- Joint network and channel coding (functional decoding)

$$C_{joint} = \tfrac{1}{4}\log_2(1 + 2 \cdot \text{SNR})$$

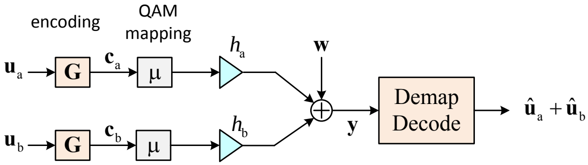
Capacities and Achievable Rates



[Wilson2010]

Application of Linear Channel Codes

- Linear encoding: $\mathbf{c}_a = \mathbf{u}_a\mathbf{G}$, $\mathbf{c}_b = \mathbf{u}_b\mathbf{G}$
- Sum of codewords: $\mathbf{c}_{ab} \triangleq \mathbf{c}_a + \mathbf{c}_b = (\mathbf{u}_a + \mathbf{u}_b)\,\mathbf{G} = \mathbf{u}_{ab}\mathbf{G}$
- Modulation (mapping): $\mathbf{x}_a = \mu(\mathbf{c}_a)$, $\mathbf{x}_b = \mu(\mathbf{c}_b)$
- Received signal: $\mathbf{y} = h_a\mathbf{x}_a + h_b\mathbf{x}_b + \mathbf{w}$



Decoding of Linear Channel Codes at Relay

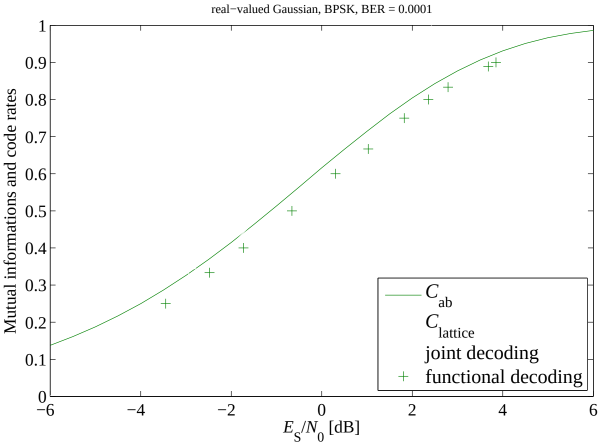
1. **Separate decoding** for $\mathbf{u}_\mathrm{a}$, $\mathbf{u}_\mathrm{b}$: compute L-values for $c_{\mathrm{a},n}$ and $c_{\mathrm{b},n}$, decode each message separately, then combine to $\hat{\mathbf{u}}_\mathrm{ab} = \hat{\mathbf{u}}_\mathrm{a} + \hat{\mathbf{u}}_\mathrm{b}$
2. **Functional decoding**: find L-values for $c_{\mathrm{ab},n}$, decoder output is $\hat{\mathbf{u}}_\mathrm{ab}$
   - Possible due to $\mathbf{c}_\mathrm{ab} = \mathbf{u}_\mathrm{ab}\mathbf{G}$
3. **Joint decoding**: find *a posteriori probability* for joint codeword symbols $d_n \triangleq 2c_{\mathrm{a},n} + c_{\mathrm{b},n}$, apply joint decoding [Wübben2010, Pfl2011, Pfl2014], then combine $\hat{\mathbf{u}}_\mathrm{ab} = \hat{\mathbf{u}}_\mathrm{a} + \hat{\mathbf{u}}_\mathrm{b}$
   - may work even if individual messages cannot be decoded

Simulation for Functional and Joint Decoding

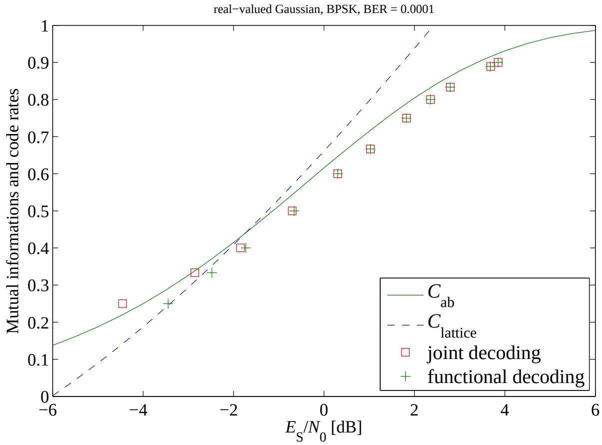
- BSPK over AWGN two-way relay channel
- Channel code familiy: DVB-S2, codeword lengths N=64800 bits, code rates $R_\mathrm{c} \in \{1/4, 1/3, 2/5, 1/2, 3/5, 2/3, 3/4, 4/5, 5/6, 8/9, 9/10\}$
- Functional decoding with standard BP decoder on L-values for $c_{\mathrm{ab},n}$
- Joint decoding with non-binary LDPC decoder [Declercq2007,Pfl2014]
- Achievable rate with nested lattice coding: $C_\mathrm{lattice} = \frac{1}{2}\log_2\left(\frac{1}{2} + \frac{2E_\mathrm{S}}{N_0}\right)$
  - is optimum for high SNR
- Mutual information between received signal and desired sum of bits:

$$C_\mathrm{ab} \triangleq I(c_\mathrm{ab}; y) = \sum_{c_\mathrm{ab}=0}^{1} \int_{-\infty}^{\infty} p(c_\mathrm{ab}, y) \log_2 \frac{p(c_\mathrm{ab}, y)}{p(c_\mathrm{ab})\, p(y)} \mathrm{d}y$$

Simulation Results



Simulation Results



Observations

- Joint decoding is always superior to functional and to separate decoding, in some cases the performance gap is large.
- Uplink of two-way relay is a special case of multiple-access channel.
  - MAC capacity region is achieved by random codebooks and successive interference cancellation
  - However, for considered cases, linear codes are superior to random codes.
- Open question: Capacity of virtual channel between c_{ab} and y?

Observations

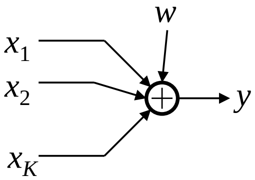
- Joint decoding is always superior to functional and to separate decoding, in some cases the performance gap is large.
- Uplink of two-way relay is a special case of multiple-access channel.
  - MAC capacity region is achieved by random codebooks and successive interference cancellation
  - However, for considered cases, linear codes are superior to random codes.
- Open question: Capacity of virtual channel between c_{ab} and y?

Uncoordinated Multiple Access

- Multiple users access the same access point without coordination
- Use cases:
  - large user population, sporadic transmissions
  - short packets (size similar to control information), "small data"
  - typical for machine-type communication
  - relevant for satellite uplink due to large round-trip delay
- Some uncoordinated multiple access schemes:
  1. Slotted ALOHA
  2. Contention Resolution Diversity Slotted ALOHA (CRDSA) [Casini2007]
  3. Irregular Repetition Slotted ALOHA (IRSA) [Liva2011]
  4. Frameless ALOHA

The Gaussian Multiple-Access Channel

- K users, AWGN $w \sim \mathcal{CN}(0,1)$
- each user transmits with power $P_k = \mathbb{E}\left[|x_k|^2\right]$



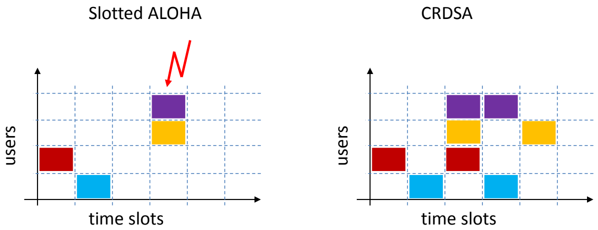
$$y = \sum_{k=1}^{K} x_k + w$$

- Capacity region:

$$\mathcal{C}_{\text{MAC}} = \left\{ \mathbf{R} : \sum_{k \in \mathcal{K}} R_k \leq \log_2 \left(1 + \sum_{k \in \mathcal{K}} P_k \right) \; \forall \mathcal{K} \in \{1, \ldots, K\} \right\}$$

Slotted ALOHA and CRDSA

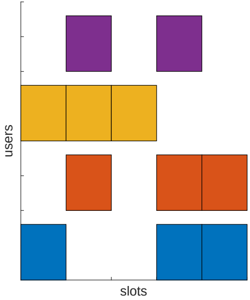
- Slotted ALOHA suffers from packet collisions
- CRDSA transmits each packet twice and applies successive interference cancellation (SIC) on a packet basis

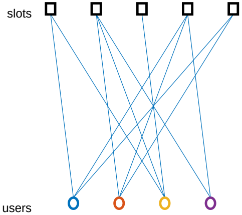


- IRSA transmits random number of packet replicas according to an optimized distribution

Irregular Repetition Slotted ALOHA

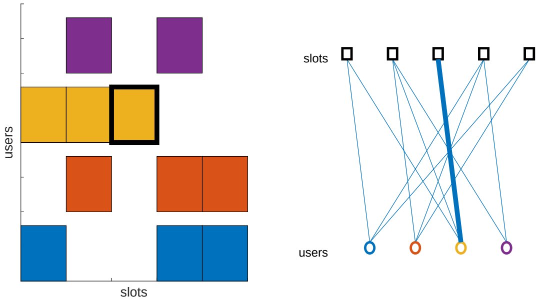
- Related to sparse graph codes
  - Packet replicas can represented by a Tanner graph
  - SIC is equivalent to belief propagation for erasure channel





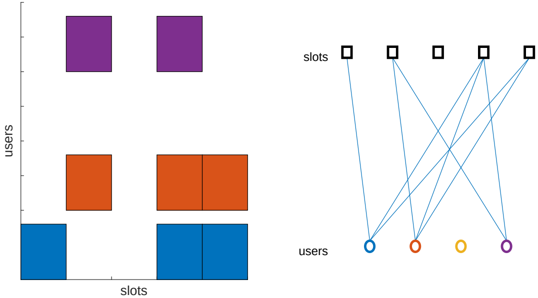
Irregular Repetition Slotted ALOHA

- Related to sparse graph codes
  - Packet replicas can represented by a Tanner graph
  - SIC is equivalent to belief propagation for erasure channel



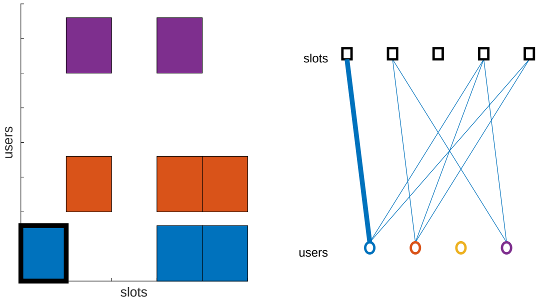
Irregular Repetition Slotted ALOHA

- Related to sparse graph codes
  - Packet replicas can represented by a Tanner graph
  - SIC is equivalent to belief propagation for erasure channel



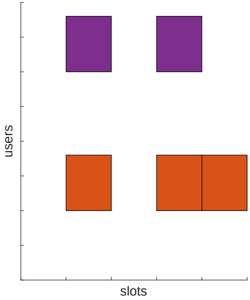
Irregular Repetition Slotted ALOHA

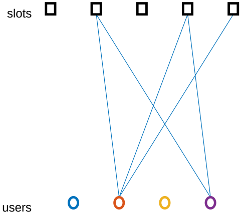
- Related to sparse graph codes
  - Packet replicas can represented by a Tanner graph
  - SIC is equivalent to belief propagation for erasure channel



Irregular Repetition Slotted ALOHA

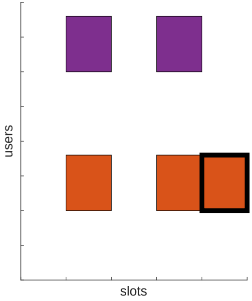
- Related to sparse graph codes
  - Packet replicas can represented by a Tanner graph
  - SIC is equivalent to belief propagation for erasure channel

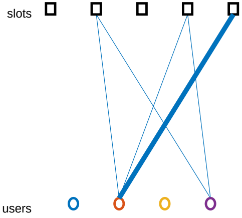




Irregular Repetition Slotted ALOHA

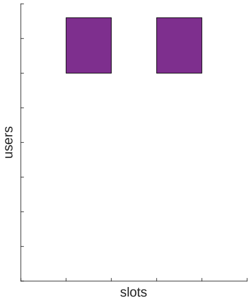
- Related to sparse graph codes
  - Packet replicas can represented by a Tanner graph
  - SIC is equivalent to belief propagation for erasure channel

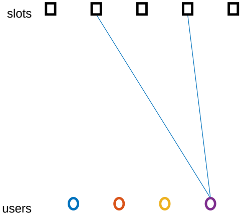




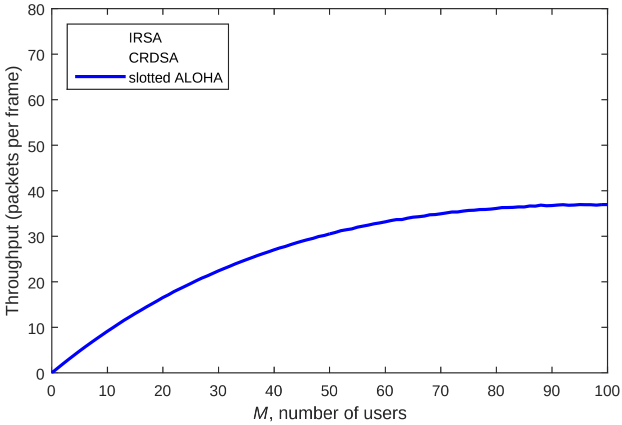
Irregular Repetition Slotted ALOHA

- Related to sparse graph codes
  - Packet replicas can represented by a Tanner graph
  - SIC is equivalent to belief propagation for erasure channel

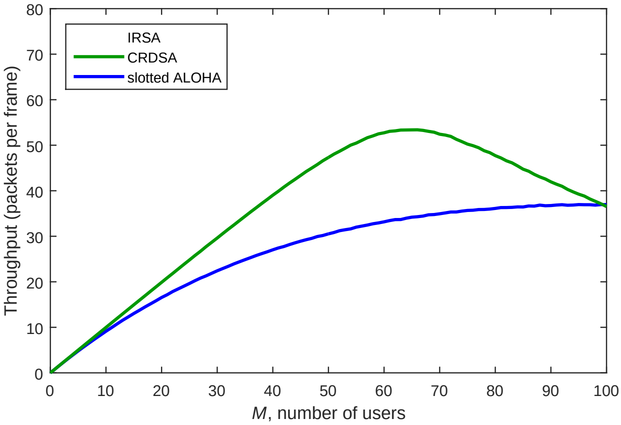




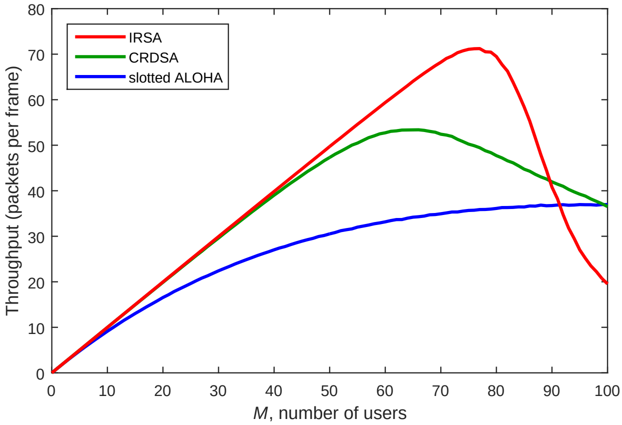
Throughput with Slotted ALOHA and its Evolutions



Throughput with Slotted ALOHA and its Evolutions



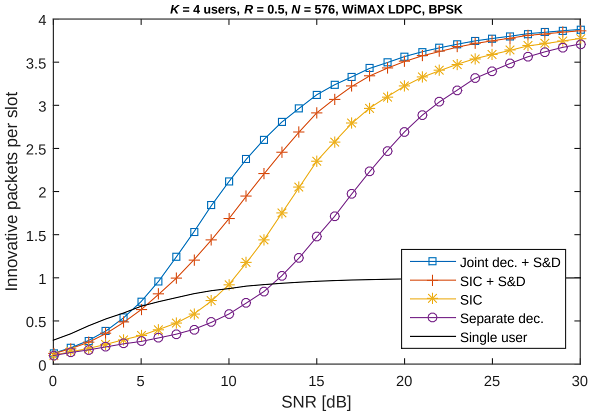
Throughput with Slotted ALOHA and its Evolutions



Network Coding for Multiple Access: Seek & Decode, Joint Decoding

- Concepts of physical-layer network coding can be applied to packet collisions on a slot-by-slot basis
    1. Separate decoding of each packet in collision: packets may be decodable despite of collisions ("capture effect")
    2. After one packet has been decoded in a collision, interference cancellation can be applied
    3. Network (de)coding: decode a sum of collided packets ("Seek & Decode", [Cocco2014])
    4. Joint decoding of collided packets

Simulation Results



Summary

- Network coding
  - Applicable on many layers: from physical to application layer
  - Unique in engineering: no analogy to traffic, fluids, etc.
- Uncoordinated multiple access
  - Relevant for machine-type communication for small data
  - Concepts of physical-layer network coding are applicable

DLR

Some References I

- [Ahlswede2000] R. Ahlswede, N. Cai, S.-Y. R. Li, R. W. Yeung, "Network Information Flow", *IEEE Transactions on Information Theory*, July 2000.
- [Casini2007], E. Casini, R. De Gaudenzi, O. del Rio Herrero, "Contention Resolution Diversity Slotted ALOHA (CRDSA): An Enhanced Random Access Scheme for Satellite Access Packet Networks", *IEEE Trans. Wireless Commun.*, April 2007.
- [Cocco2014] G. Cocco, S. Pfletschinger, "Seek and decode: Random multiple access with multiuser detection and physical-layer network coding", *IEEE ICC Workshop on Massive Uncoordinated Access Protocols (MASSAP)*, June 2014.
- [Declercq2007] D. Declercq, M. Fossorier, "Decoding algorithms for nonbinary LDPC codes over GF(q)", *IEEE Trans. Commun.*, April 2007.
- [Liva2011] G. Liva, "Graph-Based Analysis and Optimization of Contention Resolution Diversity Slotted ALOHA", *IEEE Trans. Commun.*, Feb. 2011.
- [Nazer2011] B. Nazer, M. Gastpar, "Reliable physical layer network coding", *Proc. IEEE*, March 2011.
- [Pfl2011] S. Pfletschinger, "Practical physical-layer network coding scheme for the uplink of the two-way relay channel", *Asilomar Conference on Signals, Systems, and Computers*, Nov. 2011.

DLR

Some References II

- [Pfl2014] S. Pfletschinger, "Joint decoding of multiple non-binary codewords", *IEEE ICC Workshop on Massive Uncoordinated Access Protocols (MASSAP)*, June 2014.
- [Sykora2011] J. Sykora, A. Burr, "Wireless Network Coding: Network Coded Modulation in the Network Aware PHY Layer", *Tutorial at WCNC* 2011.
- [Wilson2010] M.P. Wilson, K. Narayanan, H. D. Pfister, A. Sprintson, "Joint physical layer coding and network coding for bi-directional relaying", *IEEE Trans. on Information Theory*, Nov. 2010.
- [Wübben2010] D. Wübben, Y. Lang, "Generalized sum-product algorithm for joint channel decoding and physical-layer network coding in two-way relay systems", *IEEE Globecom*, Dec. 2010.