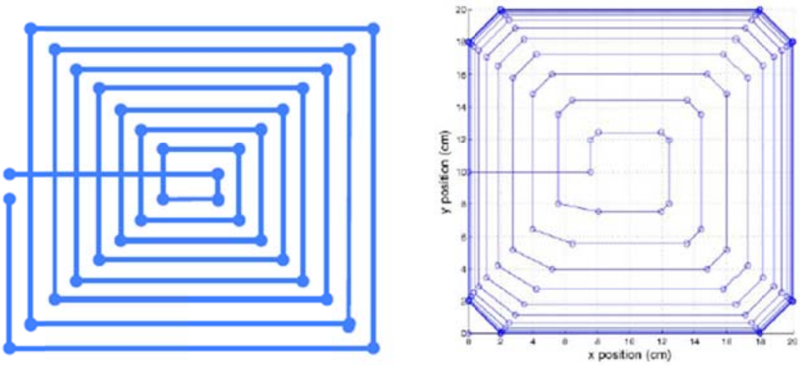
# Theoretical assignment

**Question 1** - **Wireless Power Transfer**
Explain reasons why a single shunt rectifier with one diode a) can achieve theoretically 100% efficiency and b) higher efficiency than that of charge pump and bridge full-wave rectifier.

**Question 2 - Wireless Power Transfer: From Far Field to Near Field**
In near-field magnetic-coupling wireless power transfer, transmitter coil design plays an important role to achieve evenly distributed magnetic field and a power transfer efficiency less sensitive to the location of receiver coil. For planar magnetic-coupling wireless power transfer systems, which of the following two transmitter coil designs will achieve a more uniform magnetic field distribution across its surface area? (a) Coil with equal space between adjacent turns, (b) Coil with increased space between adjacent turns toward center.




(a) (b)

**Question 3 – Paper writing**

a) If you are writing a paper on the development of a wireless power system and want to publish in an MTT-S journal, is experimental data required?
b) If you do not have experimental confirmation of your system, suggest another journal?

**Question 4 – Passive radio communications combining backscatter with WPT**
How can we measure the coverage range of back-scatter radios?

**Question 5 – UWB-UHF circuit and systems**
What are the pros and cons of merging UWB and UHF for next generation RF-ID?

**Question 6 – Energy harvesting**

a) What is the maximum theoretical efficiency of an ideal (single energy gap) solar cell, and what is the maximum reported efficiency of crystalline silicon solar cells?

b) Consider a UHF (868 MHz) RFID reader which transmits 1 W and has an antenna gain of 6 dB. Assuming an RFID tag with antenna gain of 0 dB and sensitivity of -20 dBm, estimate the maximum distance (range) that the tag can be detected?

**Question 7 – Strategies for energy harvesting communication networks**

a) Assume that an energy harvesting communication device harvest 1, 2, and 3 μJ (micro Joules) at time t=0, 1 and 2 seconds, respectively. If the rate of transmission, R, depends on the transmission power, P, through the relation $R = a\sqrt{P}$ bits per second, find the optimal power allocation for this system from t=0 until t=3.

b) Now consider there is fading in the system, and the transmission rate is given by $R = \log(1 + h \cdot P)$, where $h$ is the channel gain. If the channel state is constant within each unit time slot, and it is getting worse as time goes by, what would be the optimal transmission power over the time period t=0 to t=3?

How would the power allocation change if the channel conditions were worse initially, and it improves over time. Explain the impact of finite battery capacity on the optimal solution.

**Question 8 – Coded cooperation**

The outage probability is defined as the probability that the rate exceeds channel capacity

$$P_{\text{outage}} = P_r(C(\gamma) < R)$$

Recall that the capacity for the AWGN channel is given by

$$C(\gamma) = \log_2(1 + \gamma)$$

where $\gamma$ denotes the SNR. Then, For a Rayleigh fading channel with average SNR $\bar{\gamma}$ the outage probability is given by

$$P_{\text{outage}} = P_r(\gamma < 2^R - 1) = 1 - \exp\left(-\frac{2^R-1}{\bar{\gamma}}\right)$$

The exercise is for you to sketch the outage probability for the coded cooperation scheme with 2 users, expressing the outage probability of each disjoint event. That is, it is assumed the four cases are disjoint (with mutually independent SNRs). Hence,

$$P_{\text{outage}}{}^{(u)} = P_{\text{outage},1}{}^{(u)} + P_{\text{outage},2}{}^{(u)} + P_{\text{outage},3}{}^{(u)} + P_{\text{outage},4}{}^{(u)}$$

The four events are specified below. The solution is given for the first one ($\Omega = 1$).
It is also assumed both users applies the same cooperation rate $\alpha$, where $\alpha = N_1/N$ is the portion of the codeword transmitted during the broadcast phase (phase 1) and $1 - \alpha = N_2/N$ the portion of the codeword transmitted during the multiple access phase (phase 2), with $N = N_1 + N_2$ the total codeword length.
Recall that if one user, $u = 1$, fails to decode the other user, $u = 2$, message it will transmit its own additional parity bits in the multicast phase, $c_2^{(1)}$, whereas if it decodes it correctly, it will send the other user additional parity bits, $c_2^{(2)}$.

i. **Event $\Omega = 1$: Both users are able to decode correctly**

*Solution:*

*Notation: $c_1^{(1)}$ user 1 first part of codeword (N1 bits); $c_2^{(1)}$ user 1 second part of codeword (N2 bits); $c_1^{(2)}$ user 2 first part of codeword (N1 bits); $c_2^{(2)}$ user 1 second part of codeword (N2 bits);*

***Broadcast phase**: User 1 transmits $c_1^{(1)}$ and User 2 transmits $c_1^{(2)}$*

*if both users are able to decode correctly it means there is no outage, hence the rate is below capacity*

*In particular:*

*The capacity of the channel from user 1 to user 2 is*

$$C_{12}(\gamma_{12}) = \log_2(1+\gamma_{12}) > \frac{R}{\alpha}$$

*Equivalently, the capacity of the channel from user 1 to user 2 is*

$$C_{21}(\gamma_{21}) = \log_2(1+\gamma_{21}) > \frac{R}{\alpha}$$

***Multicast phase**: User 1 transmits $c_2^{(2)}$ and User 2 transmits $c_2^{(1)}$*

*the two transmissions can be viewed as parallel Gaussian channels (conditioned on the previous events in broadcast phase) => capacities added. Then*

$$C_{1d}(\gamma_{1d},\gamma_{2d}|\Omega = 1) = \alpha C_{1d}(\gamma_{1d}) + (1-\alpha)C_{2d}(\gamma_{2d}) < R$$

*and*

$$C_{2d}(\gamma_{1d},\gamma_{2d}|\Omega = 1) = \alpha C_{2d}(\gamma_{2d}) + (1-\alpha)C_{1d}(\gamma_{1d}) < R$$

*(note that the final rate is $R$ )*

$$C_{1d}(\gamma_{1d},\gamma_{2d}|\Omega = 1) = \alpha\log_2(1+\gamma_{1d}) + (1-\alpha)\log_2(1+\gamma_{2d}) < R$$
$$C_{2d}(\gamma_{1d},\gamma_{2d}|\Omega = 1) = \alpha\log_2(1+\gamma_{2d}) + (1-\alpha)\log_2(1+\gamma_{1d}) < R$$

*Then the outage probability first component for user 1 is*

$$P_{outage,1}{}^{(1)} = P_r\left(C_{12}(\gamma_{12}) > \frac{R}{\alpha}\right)P_r\left(C_{21}(\gamma_{21}) > \frac{R}{\alpha}\right)P_r(C_{1d}(\gamma_{1d},\gamma_{2d}|\Omega = 1) < R)$$
$$= P_r\left(\gamma_{12} > 2^{\frac{R}{\alpha}} - 1\right)P_r\left(\gamma_{12} > 2^{\frac{R}{\alpha}} - 1\right)P_r\left((1+\gamma_{1d})^{\alpha}(1+\gamma_{2d})^{(1-\alpha)} < 2^R\right)$$

*For user 2 would be*

$$P_{outage,1}{}^{(2)} = P_r\left(C_{21}(\gamma_{21}) > \frac{R}{\alpha}\right)P_r\left(C_{12}(\gamma_{12}) > \frac{R}{\alpha}\right)P_r(C_{2d}(\gamma_{1d},\gamma_{2d}|\Omega = 1) < R)$$

ii. **Event $\Omega = 2$: Neither both users are able to decode correctly**

Solution:

***Broadcast phase**: User 1 transmits $c_1^{(1)}$ and User 2 transmits $c_1^{(2)}$*

*if both users are not able to decode correctly it means there is an outage, hence the rate is above capacity*

*In particular:*

*The capacity of the channel from user 1 to user 2 is*

$C_{12}(\gamma_{12}) = \log_2(1+\gamma_{12}) < \frac{R}{\alpha}$

*Equivalently, the capacity of the channel from user 1 to user 2 is*

$C_{21}(\gamma_{21}) = \log_2(1+\gamma_{21}) < \frac{R}{\alpha}$

***Multicast phase***: *User 1 transmits* $c_2^{(1)}$ *and User 2 transmits* $c_2^{(2)}$

$$C_{1d}(\gamma_{1d}, \gamma_{2d} | \Omega = 2) = \alpha \log_2(1+\gamma_{1d}) + (1-\alpha)\log_2(1+\gamma_{1d}) < R$$

$$P_{outage,2}{}^{(1)} =$$

iii. **Event $\Omega = 3$: User 2 decodes User 1 message but User 1 fails to decode User 2 message**

***Broadcast phase***: *User 1 transmits* $c_1^{(1)}$ *and User 2 transmits* $c_1^{(2)}$

***Multicast phase***: *User 1 transmits* $c_2^{(1)}$ *and User 2 transmits* $c_2^{(1)}$

$$P_{outage,3}{}^{(1)} =$$

iv. **Event $\Omega = 4$: User 1 decodes User 2 message but User 2 fails to decode User 1 message**

***Broadcast phase***: *User 1 transmits* $c_1^{(1)}$ *and User 2 transmits* $c_1^{(2)}$

***Multicast phase***: *User 1 transmits* $c_2^{(2)}$ *and User 2 transmits* $c_2^{(2)}$

$$P_{outage,4}{}^{(1)} =$$