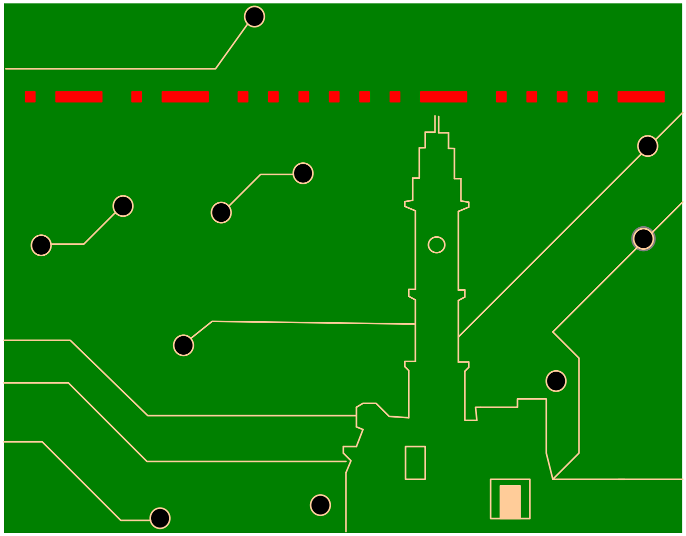
ΤΗΛ412 Ανάλυση & Σχεδίαση (Σύνθεση) Τηλεπικοινωνιακών Διατάξεων

Διάλεξη 3



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ΗΜΜΥ Πολυτεχνείου Κρήτης, Φθινόπωρο 2014

Lecture 3 – Basic Concepts (cont’d)

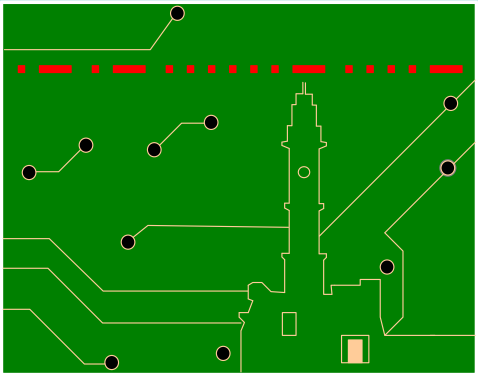
Basic Unknown Concepts:

- Previous lecture: Introduction to nonlinearities

(1-dB CP, IP3)

Today:

- Thermal noise of a resistor
- Calculating Noise Figure (NF)
- Notion of Sensitivity
- Notion of Dynamic Range



Διάλεξη 3

Most Figures for today’s lecture come from:
B. Razavi, RF Microelectronics, Prentice Hall 1998.

Thermal noise voltage of a resistor

Assume resistor R, then thermal noise induced voltage across the resistor:

$\overline{V_n^2} = 4\ k\ T\ R$ = average squared voltage for unit bandwidth (V^2/Hz)

k = 1.38 x 10^{-23} Joules/K (Boltzmann constant)
T = absolute temperature (in Kelvin)

- Remark 1: stems from PSD =*2 k T R* of two-sided thermal noise which can be considered WHITE up to |f|<100 GHz

- Remark 2: factor of 2 in voltage-squared above stems for considering both positive as well as negative freqs!

- Caution: PSD has units of power per unit bandwidth (Watt/Hz) but $\overline{V_n^2}$ has units of V^2/Hz [try BW=100MHz, R=1MOhm and test oscilloscope]

Remember Noise Figure?

$$\text{noise figure} = \frac{SNR_{\text{in}}}{SNR_{\text{out}}}$$

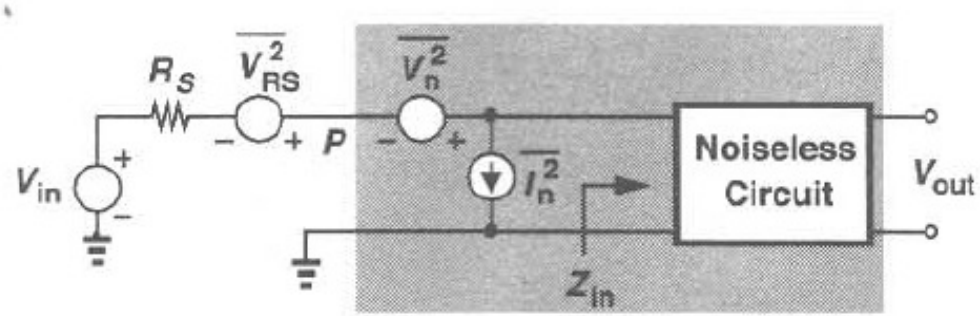
- Noiseless system (ideal) => NF = 1 (0 dB).
- Noise Figure of Cascaded Systems (Friis eq.):

$$NF_{\text{tot}} = 1 + (NF_1 - 1) + \frac{NF_2 - 1}{A_{p1}} + \cdots + \frac{NF_m - 1}{A_{p1} \cdots A_{p(m-1)}}$$

Power gain (not voltage gain)

- NF of a stage decreases with gain of previous stage =>
- …initial stages are the most (NF)-critical!

How do we calculate Noise Figure?



- Basic principle: we refer SNRs to input source resistance R_s.

…assuming VOLTAGE gain α from V_{in} to P

and A_υ from P to V_{out}

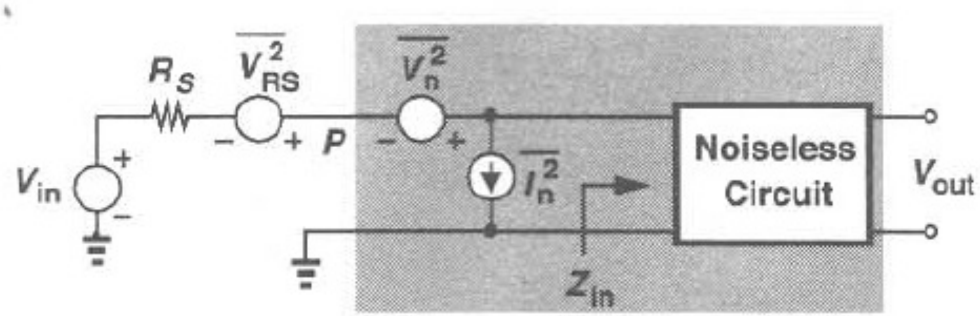
$$SNR_{in} = \frac{\alpha^2 V^2_{in}}{\alpha^2 \overline{V^2_{RS}}}.$$

$$SNR_{out} = \frac{\alpha^2 A^2_\upsilon V^2_{in}}{[\overline{V^2_{RS}} + \overline{(V_n + I_n R_S)^2}]\alpha^2 A^2_\upsilon}$$

$$= \frac{V^2_{in}}{[\overline{V^2_{RS}} + \overline{(V_n + I_n R_S)^2}]},$$

Refer internal noise at the device input!

How do we calculate Noise Figure?



- Basic principle: we refer SNRs to input source resistance R_s.

…assuming VOLTAGE gain α from V_{in} to P
and A_υ from P to V_{out}

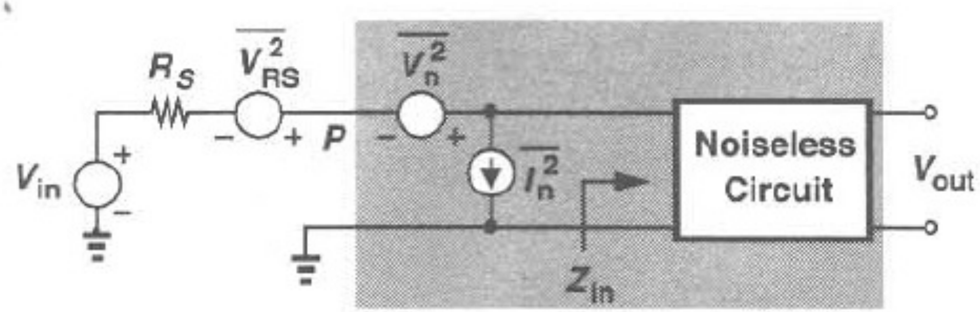
$$\text{noise figure} = \frac{SNR_{in}}{SNR_{out}} = \frac{\overline{V_{RS}^2} + \overline{(V_n + I_n R_S)^2}}{\overline{V_{RS}^2}}$$

$$= 1 + \frac{\overline{(V_n + I_n R_S)^2}}{\overline{V_{RS}^2}}$$

(Per unit bandwidth)

$$NF = 1 + \frac{\overline{(V_n + I_n R_S)^2}}{4kTR_S}$$

How do we calculate Noise Figure?



$$NF = 1 + \frac{\overline{(V_n + I_n R_S)^2}}{4kTR_S}$$

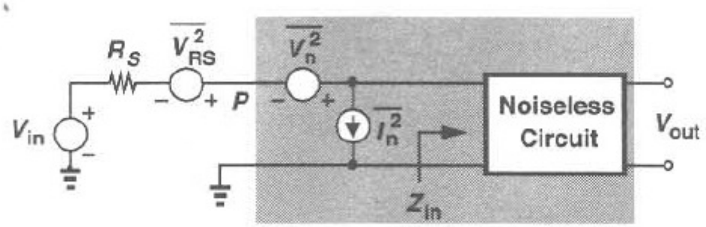
$$A = aA_{\upsilon}$$

$$\begin{aligned} NF &= \frac{4kTR_S + \overline{(V_n + I_n R_S)^2}}{4kTR_S} \\ &= \frac{A^2[4kTR_S + \overline{(V_n + I_n R_S)^2}]}{A^2}\frac{1}{4kTR_S} \\ &= \frac{V^2_{n,out}}{A^2}\frac{1}{4kTR_S}, \end{aligned}$$

Total measured noise at device output

How do we calculate Noise Figure?

$$
\begin{aligned}
NF &= \frac{4kTR_S + \overline{(V_n + I_n R_S)^2}}{4kTR_S} \\
&= \frac{A^2[4kTR_S + \overline{(V_n + I_n R_S)^2}]}{A^2} \frac{1}{4kTR_S} \\
&= \frac{V_{n,out}^2}{A^2} \frac{1}{4kTR_S},
\end{aligned}
$$



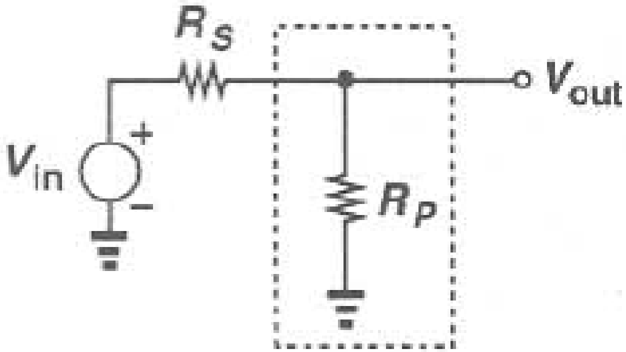
$$A = aA_\upsilon$$

- Remark 1: NF is defined according to source (input) resistance!

…which means that in cascaded systems, the output resistance of a preceding stage is needed to calculate the NF of the immediately next stage
(more on this later).

- Remark 2: end-to-end voltage gain A (squared) is also needed!

Example 1:



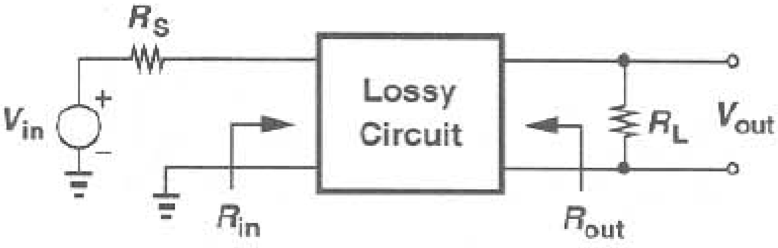
$$V_{n,\text{out}}^2 = 4kT(R_S||R_P),$$

$$A_v = \frac{R_P}{R_S + R_P}. \Rightarrow NF = 4kT(R_S||R_P)\frac{(R_S + R_P)^2}{R_P^2}\frac{1}{4kTR_S}$$

$$= 1 + \frac{R_S}{R_P}.$$

- Condition for minimum NF does not coincide with maximum power transfer.

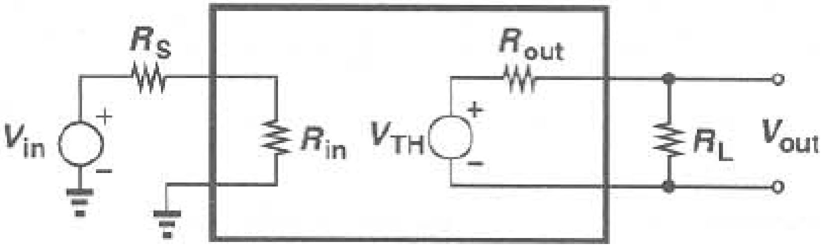
Example 2 (Lossy circuit):



(a)

$$L = \frac{P_{in}}{P_{out}} \qquad P_{in} \propto \frac{V_{in}^2}{R_s} \qquad P_{out} \propto \frac{V_{TH}^2}{R_{out}}$$

$$L = \frac{V_{\mathrm{in}}^2}{V_{TH}^2}\frac{R_{\mathrm{out}}}{R_S}$$

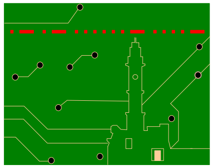


$$V_{n,\mathrm{out}}^2 = 4kTR_{\mathrm{out}}\frac{R_L^2}{(R_L + R_{\mathrm{out}})^2}$$

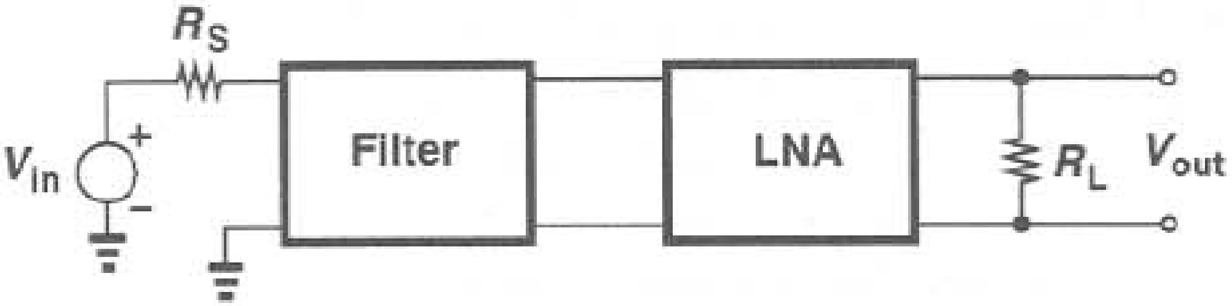
$$A_v = \frac{V_{TH}}{V_{\mathrm{in}}}\frac{R_L}{R_L + R_{\mathrm{out}}}$$

$$NF = 4kTR_{\mathrm{out}}\frac{V_{\mathrm{in}}^2}{V_{TH}^2}\frac{1}{4kTR_S} = L.$$

- Power loss = NF for Lossy circuits (!!!)
- ATTENTION: L is power LOSS (not GAIN), i.e L^{-1} = G



Example 3: series of Filter and LNA



$$
\begin{aligned}
NF_{tot} &= NF_{filt} + \frac{NF_{LNA} - 1}{L^{-1}} \\
&= L + (NF_{LNA} - 1)L \\
&= L \cdot NF_{LNA},
\end{aligned}
$$

- WARNING: the above assumes that NF_{LNA} is referenced to input resistance equal to the output resistance of the above Filter…
- Usually in RF engineering, all systems are designed around 50 (75) Ωhm.
- Don’t forget: Friss equation utilizes power gains (not voltage gains).

Notion of Sensitivity:

“the minimum signal level that the system can detect with acceptable signal-to-noise ratio”.

$$NF = \frac{SNR_{\text{in}}}{SNR_{\text{out}}} = \frac{P_{\text{sig}}/P_{RS}}{SNR_{\text{out}}} \Rightarrow P_{\text{sig}} = P_{RS} \cdot NF \cdot SNR_{\text{out}}$$

(Signal power per unit of BW)

$$\Downarrow$$

$$P_{\text{sig,tot}} = P_{RS} \cdot NF \cdot SNR_{\text{out}} \cdot B$$

$$\Downarrow$$

$$P_{\text{in,min}}|_{\text{dBm}} = P_{RS}|_{\text{dBm/Hz}} + NF|_{\text{dB}} + SNR_{\text{min}}|_{\text{dB}} + 10\log B$$

$$P_{RS} = \frac{4kTR_S}{4}\frac{1}{R_{\text{in}}} = kT = -174\ \text{dBm/Hz}$$

Why 4?
÷ 2: rms value
÷ 2: power matching

Notion of Sensitivity:

“The minimum signal level that the system can detect with acceptable signal-to-noise ratio”.

$$P_{\text{in,min}}|_{\text{dBm}} = P_{RS}|_{\text{dBm/Hz}} + NF|_{\text{dB}} + SNR_{\text{min}}|_{\text{dB}} + 10\log B$$

$$\Downarrow$$

$$P_{\text{in, min}} = \underbrace{-174\,\text{dBm/Hz} + NF + 10\log B}_{=\text{“noise floor F”}} + SNR_{\text{min}}$$

- WARNING1: small detectable signal (high sensitivity) might be the result of small communication bandwidth!
- WARNING2: SNR_{min} is the output, operational SNR.

Notion of Dynamic Range:

"The ratio of the maximum input level the system tolerates to the minimum signal level the system provides a reasonable signal quality".

- notice that the definition is rather vague => different metric in ADCs, different metric in RF systems…

- RF systems: min signal according to sensitivity…
  max signal according to intermodulation behavior,
  i.e. input level where IM3 products equal noise floor.

=> "Spurious-free dynamic range (SFDR)"

$$P_{IIP3} = P_{\text{in}} + \frac{P_{\text{out}} - P_{IM,\text{out}}}{2}$$

Notion of Dynamic Range:

$$P_{IIP3} = P_{\mathrm{in}} + \frac{P_{\mathrm{out}} - P_{IM,\mathrm{out}}}{2}$$

$$\Downarrow$$

$$P_{IIP3} = P_{\mathrm{in}} + \frac{P_{\mathrm{in}} - P_{IM,\mathrm{in}}}{2} = \frac{3P_{\mathrm{in}} - P_{IM,\mathrm{in}}}{2},$$

$$\Downarrow$$

$$P_{\mathrm{in}} = \frac{2P_{IIP3} + P_{IM,\mathrm{in}}}{3}$$

$$\Downarrow$$

$$P_{\mathrm{in,max}} = \frac{2P_{IIP3} + F}{3}$$

$$P_{out} - P_{IM,out} = P_{in} - P_{IM,in}$$

(remember geometric proof of IIP3!)

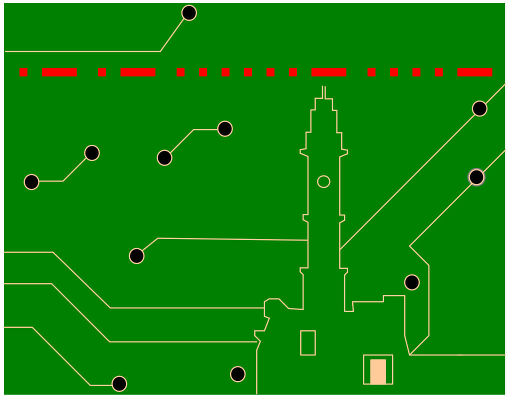
$$SFDR = \frac{2P_{IIP3} + F}{3} - (F + SNR_{\mathrm{min}}) = \frac{2(P_{IIP3} - F)}{3} - SNR_{\mathrm{min}}.$$

Example: IIP3=-15 dBm, NF=9 dB, B=200kHz, SNR_{min}=12 dB

=> SFDR ≈ 53 dB

Think of the maximum relative level of interferer that a rec can tolerate for small desired input signal!

Questions?



Next lecture: Receiver Architectures!